9

Helping Students Master the Basic Facts

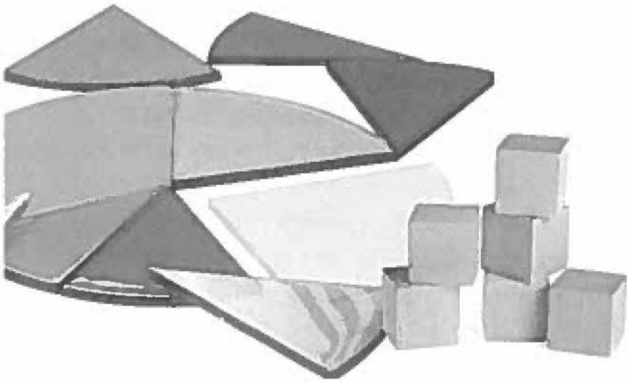
Big IDEAS

1 Number relationships provide the basis for strategies that help students learn basic facts. For example, when solving 7×8 you can help students think about decomposing the 7 into $2 + 5$. Then $(2 + 5) \times 8$ is the same as $2 \times 8 + 5 \times 8$. Using the distributive property and building off the benchmark of 5 allows students to use the structure and number relationships that continue to help them with larger numbers.

2 All of the facts are conceptually related so students can figure out new or unknown facts using those they already know. For example, "think addition" is a powerful way to think of subtraction facts. Rather than 13 "take away 6," which requires a lot of counting, students can think 6 and what adds to 13. They might add up to 10 (4) and then add 3 more to get 7, or they may think double 6 is 12 so it must be one more or 7. Or in the case of multiplication, 6×8 can be thought of as five 8s (40) and one more 8. It might also be three 8s doubled.

3 Because mastering the basic facts is a developmental process, students move through phases, starting with counting, then more efficient reasoning strategies, and eventually quick recall and mastery. Instruction must help students through these three phases without rushing them to know their facts only through memorization.

4 When students struggle with developing basic fact fluency, they may need to return to foundational ideas. Just providing additional drill will not resolve their challenges and can negatively affect their confidence and success in mathematics.



Basic facts for addition and multiplication are the number combinations for which both *addends*, or both *factors*, are less than 10. Basic facts for subtraction and division are the corresponding combinations. Therefore, $15 - 8 = 7$ is a basic subtraction fact because the corresponding addends are less than 10.

Mastery of a basic fact means that a student can give a quick response (in about 3 seconds) without resorting to inefficient means, such as counting by ones. According to the *Curriculum Focal Points* (NCTM, 2006) and the *Common Core State Standards for Mathematics* (CCSSO, 2010), addition and subtraction concepts should be learned in first grade, with quick recall of basic addition and subtraction facts mastered by the end of grade 2.

According to *Curriculum Focal Points*, concepts of multiplication and division should be learned in third grade, with quick recall of the one-digit facts (up through 9 × 9) mastered in grade 4. In the *Common Core State Standards*, however, the one-digit multiplication facts are to be known by quick recall by the end of grade 3.

Developing quick and accurate recall with the basic facts is a developmental process. It is critical that students know their basic facts well, and teaching them effectively requires much more than flash cards and timed tests. This chapter explains strategies for helping students learn their facts, including instructional approaches to use—and others to avoid. The key point: Focus on number sense! Research indicates that early number sense predicts school mathematics success more than other measures of cognition like reading ability or verbal, spatial, or memory skills (Jordan, Kaplan, Locuniak, & Ramineni, 2007; Locuniak & Jordan, 2008; Mazzocco & Thompson, 2005).

Developmental Nature of Basic Fact Mastery

Even though students in grades 4 to 10 have had ample opportunities to learn their facts, every teacher in those grades knows students who still count on their fingers, make marks in the margins to count on, or simply guess at answers. These students have not mastered their facts because they have not developed efficient methods of producing a fact answer based on number relationships and reasoning. Drilling inefficient methods does not produce mastery!

When teaching basic facts, teachers need to attend to the essential understanding that students progress through stages that will eventually result in "just knowing" that 2 + 7 is 9 or that 5 × 4 is 20. Arthur Baroody, a mathematics educator who does research on basic facts, describes three phases in the process of learning facts (2006, p. 22):

1. *Counting strategies*—using object counting (e.g., blocks or fingers) or verbal counting to determine the answer. Example: 4 + 7 = ________ . Student starts with 7 and counts on verbally (8, 9, 10, 11), or the student counts 4, counts 7, and then "counts all" over again to reach 11.
2. *Reasoning strategies*—using known information to logically determine an unknown combination. Example: 4 + 7. Student knows that 3 + 7 is 10, so 4 + 7 is one more, 11.
3. *Mastery*—producing answers efficiently (fast and accurately). Example: 4 + 7. Student quickly responds, "It's 11; I just know it."

Over many years, research supports the notion that basic fact mastery is dependent on the development of reasoning strategies (Baroody, 2003, 2006; Brownell & Chazal, 1935; Carpenter & Moser, 1984; Fuson, 1992; Henry & Brown, 2008). This chapter will focus on effective ways to teach students in grades 3 through 5 to use reasoning strategies and sense making to master the basic facts (phases 2 and 3).

Formative Assessment Note

When are students ready to work on reasoning strategies? Based on the research, they are ready to apply reasoning strategies when they efficiently use counting strategies (start with the largest number and count up, or in the case of multiplication, start with a known fact and count up one more group) and when they are able to decompose numbers (e.g., that 6 can be decomposed into 5 + 1). Pose basic fact problems to students in a diagnostic interview to see whether they show evidence of these skills. Once they have grasped these needed skills, begin work on reasoning strategies. If they are lacking one skill, provide more experiences to develop it.

Approaches to Fact Mastery

In attempting to help students master the basic facts, three somewhat different approaches can be identified. Although these approaches are all used, not all are equally effective. First is to work on memorization of each fact in isolation. A second approach can be traced at least as far back as the 1970s. Rathmell (1978) suggests that for various groupings of basic facts, teach students a collection of strategies or thought patterns that have been found to be efficient and teachable. The third approach, *guided invention*, also focuses on using strategies to learn facts; however, students generate, or reinvent, the strategies.

Memorization

Unfortunately, some curriculums and teachers move from presenting concepts of addition and multiplication straight to memorization of facts, feeling that developing strategies is not essential in this process (Baroody, Bajwa, & Eiland, 2009). This idea that students can just store the facts when practiced extensively means that students have 100 separate addition facts (just for the various combinations of 0 through 9) and 100 separate multiplication facts that must be repeatedly memorized and practiced. They may even have to memorize subtraction and division facts separately, bringing the total to more than 300 facts! However, many fourth and fifth graders have not mastered addition and subtraction facts, and many middle school students do not know their multiplication facts. This is strong evidence that the memorization method alone simply does not work. You may be tempted to respond that you learned your facts in this manner, as did many others. However, studies as long ago as 1935 (Brownell & Chazal) concluded that students actually develop a variety of different reasoning strategies for answering basic facts apart from the large amounts of isolated drill. Unfortunately, this drill does not support an increased sophistication or refinement of these strategies. Moreover, Baroody (2006, p. 27) notes that this drill approach to basic fact instruction works against the development of the five strands of mathematics proficiency (National Research Council, 2001), pointing out the following limitations:

- *Inefficiency.* There are too many facts to memorize in isolation.
- *Inappropriate applications.* Students misapply the facts and don't check their work for reasonableness.
- *Inflexibility.* Students don't learn flexible strategies for finding the sums or products and therefore continue to count by ones.

Struggling learners and students with disabilities often have difficulty memorizing so many isolated facts and are trapped in phase 1, relying on the use of counting strategies (Mazzocco et al., 2008). In addition, drill often causes unnecessary anxiety and undermines student interest and confidence in mathematics. Connecting new knowledge to what students already know allows all students to master the basic facts. Students who rely on simply counting must learn meaningful alternative approaches that allow for the development of more complex mathematical thinking (Garza-Kling, 2011).

Explicit Strategy Instruction

For approximately three decades, basic fact instruction has focused on explicitly teaching efficient strategies that are applicable to a collection of facts. Students then practice these strategies as they are taught to them. There is strong evidence to indicate that such methods are effective (e.g., Baroody, 1985; Bley & Thornton, 1995; Fuson, 1984, 1992; Rathmell, 1978).

Rather than giving students something new to memorize, explicitly teaching strategies supports student reasoning in choosing strategies that help them get solutions without counting. However, sometimes textbooks or teachers focus on memorizing a particular strategy, connecting it only to the facts that work with the strategy. This approach doesn't work (for the same reason that memorizing isolated facts doesn't work). A recent study found that students whose teachers relied heavily on memorization of basic fact strategies, rather than ones that emphasized reasoning about the strategies, had low number-sense proficiency (Henry & Brown, 2008). This memorization approach should be avoided.

Guided Invention

The third approach (Gravemeijer & van Galen, 2003) connects fact mastery to students' collection of number relationships. Some students may think of 9 × 4 as "9 times 2 is 18 and double that for 36." Other students note that 10 × 4 is 40, so you take 4 from the 40 to get 36. Still other students may know 9 × 5 is 45 and you take 9 from that to get 36. What is important is that students use number combinations and relationships that make sense to them.

Gravemeijer and van Galen call this approach *guided invention* because many of the efficient strategies will not be developed by all students without teacher guidance. That is, you cannot simply place all of your efforts on number relationships and the meanings of the operations and assume that fact mastery will magically occur. Instead, you should design sequenced tasks and problems that will promote students' invention of effective strategies. Then, students need to clearly articulate these strategies and share them with peers. This sharing is often best carried out in think-alouds, in which students talk through the decisions they made and share counterexamples.

Facilitating Strategy Development

To guide your students to use effective strategies, you need to have knowledge of many successful approaches. With this knowledge, you will be able to recognize the emergence of effective strategies as your students develop them and at the same time help others capitalize on their peers' ideas.

Plan experiences that help students move on the trajectory from counting to strategies to mastery and quick recall. One effective approach is to use story problems with numbers selected in such a manner that students are most likely to develop a particular strategy as they solve them. In discussing student strategies, you can focus attention on the methods that are most effective.

If you look at 6 + 7, you will find some students may count on from 7 (7, 8, 9, 10, 11, 12, 13). Others will use the Up Over 10 strategy (7 to 10 is a jump of 3 and 3 more is 13). Help students who are counting on to see the connections to Up Over 10. This moves students from counting (phase 1) to reasoning strategies (phase 2). To move from reasoning strategies to recall (phase 3), continue to develop story problems that have numbers that go up over 10. Students will build up speed the more they practice this strategy. Eventually, students say "I just knew it" because these mental strategies become fluent.

Story Problems

Story problems provide a context that can help students understand the situation and apply flexible strategies for doing computation. Some teachers may express hesitancy to use story problems with ELLs or students with disabilities because of the additional language or reading required, but because language supports understanding, story problems are important for all students.

Contexts selected must be relevant and understood. Consider, for example, that the class is working on the × 3 facts. The teacher poses the following question:

In 3 weeks we will be going to the zoo. How many days until we go to the zoo?

Suppose that Aidan explains how he figured out 3 × 7 by starting with double 7 (14) and then adding 7 more. He knew that 6 added onto 14 equals 20 and one more is 21. You can ask another student to explain Aidan's thinking.

Standards for Mathematical Practice

3 Construct viable arguments and critique the reasoning of others

This requires students to attend to ideas that come from their classmates. Now explore with the class what other facts might work with Aidan's strategy. Some may notice that all of the facts with a 3 as a factor will work for the "double and add one more" strategy. Others may say that you can always add one more group on if you know the next smaller fact. For example, for 6 × 8, you can start with 5 × 8 and add 8. Students with disabilities may be challenged to keep all of their peers' ideas in working memory, so recording the ideas on the board is an effective support.

Posing a daily story problem such as the preceding one, followed by a brief discussion of the various strategies that students used, can improve students' accuracy and efficiency with basic facts (Rathmell, Leutzinger, & Gabriele, 2000). A similar approach is shown in Figure 9.1, which includes a story example intended to support reasoning strategies from grade 4 of *Investigations in Number, Data, and Space* (Russell & Economopoulos, 2008). Research has found that when a strong emphasis is placed on students solving story problems, they not only become better problem solvers but also master more basic facts than students in a drill program (National Research Council, 2001).

Reasoning Strategies

A second approach is to directly model a reasoning strategy. A lesson may be designed to have students examine a specific collection of facts for which a particular type of strategy is appropriate. You can discuss how these facts are all alike in some way, or you might suggest an approach and see whether students are able to use it with similar facts.

Continue to discuss strategies invented by students and plan lessons that encourage conversations about strategies. Don't expect to have a strategy introduced and understood with just one story problem or one exposure. Just as when using story problems, students need lots of opportunities to make the strategies their own, or to use new strategies that their classmates have shared. Many students will simply not be ready to use an idea in the first few days, and then suddenly something will connect and a useful idea will be theirs. No student should be forced to adopt someone else's strategy, but every student should be required to understand strategies that are brought to the discussion.

Figure 9.1

Story problem from the *Investigations in Number, Data, and Space* curriculum to develop basic fact reasoning strategies.

Grade 4, Unit 1: Factors, Multiples, and Arrays

Lesson: Making Arrays

A package of juice boxes has 8 juice boxes.

How many juice boxes are in 3 packages?

How many juice boxes are in 6 packages?

How many juice boxes are in 9 packages?

Source: Van de Walle, John; Karp, Karen S.; Bay-Williams, Jennifer M., *Elementary and Middle School Mathematics: Teaching Developmentally,* 8th ed., © 2013. Reprinted and electronically reproduced by permission of Pearson Education, Inc., Upper Saddle River, New Jersey. Originally in Russell, S. J., & Economopoulos, K. (2008). *Investigations in Number, Data, and Space (Grade 4).* New York: Pearson.

Reasoning Strategies for Addition Facts

Addition facts—the sums through 20—are considered mastery items in the second-grade curriculum (CCSSO, 2010). However, very few third-grade teachers will ever see a new class that has mastered all of these facts, and many teachers of fifth grade and higher have students who still have gaps.

For teachers in grades 3 through 5, the following ideas are important:

- All of the addition facts can be connected to one or more very important number relationships. For students who have not mastered addition facts, time will be saved by devoting attention to those relationships rather than to spending time on drill.
- Students will rarely master a subtraction fact without knowing the corresponding addition fact. That is, if a student knows 12 − 8, it is almost certain that he or she knows 8 + 4. Therefore, mastery of addition facts should be seen as a prerequisite to subtraction facts.
- Diagnosis of what facts individual students have mastered and where gaps remain will help you plan a targeted method to help students. Time invested in this analysis will save time in the long run.

The following sections provide a brief look at strategies for addition facts. With this information you can plan effective activities to help students in grades 3 through 5 reach mastery.

One More Than and Two More Than

As students enter third grade there is an expectation that some strategy use and facts are mastered. Because 51 of the 100 addition facts involve a 0, 1, or a 2, these are likely facts that are known. Students who are missing facts with a zero are often holding on to the faulty notion that "addition makes bigger." Therefore, they might answer 8 to 7 + 0. These facts do not require any strategy, but rather a good understanding of the meaning of zero and addition.

Many students in grades 1 and 2 have been taught to use a counting-on strategy for facts with addends of 1 or 2, as well as those involving a 3. If students are using a counting-on strategy efficiently—and using it only for these small addends—do not try to stop it. However, we strongly suggest that you discourage the use of counting on for all facts. It is difficult for some students to separate counting on for some facts and not for others. Students in grades 3 through 5 who have not yet mastered addition facts are often using counting on for facts such as 8 + 5, for which that strategy is not efficient.

Activity 9.1 ONE MORE THAN AND TWO MORE THAN WITH DICE AND SPINNERS

Make a die with sides labeled + 1, + 2, + 1, + 2, "one more," and "two more." Use with another die labeled 3, 4, 5, 6, 7, and 8 (or whatever values students need to practice). After each roll of the dice, students should say the complete fact: "Four and two more equals six." Alternatively, roll one die and use a spinner with + 1 on one half and + 2 on the other half. For students with disabilities, you may want to start with a die that just has + 1 on every side and then another day move on to a + 2 die. This will help emphasize and practice one approach.

Make 10

Perhaps the most important strategy for students to know is the Make 10 strategy, which includes the combinations that add to 10. Starting with story problems using two numbers that add to 10, or that ask how many are needed to make 10, can assist this process.

The ten-frame is a very useful tool for creating a visual image for students developing this strategy. Place six counters starting on the upper left corner on one ten-frame and ask, "How many more to make 10?" This activity can be repeated frequently with counters or the little ten-frame cards (see Blackline Masters 3 and 4) until all combinations that make 10 are mastered. Later, display a blank ten-frame to help build the visual image and say a number less than 10. Students start with that number and complete the 10 fact. If you say four, they say "Four plus six equals ten." This can also be done individually or in small groups. This knowledge supports other strategies (described later in this chapter) such as Up Over 10 in addition and Down Over 10 and Take from the 10 in subtraction.

BLM

Knowing number combinations that make 10 not only helps with basic fact mastery but also builds the foundations for working on addition with higher numbers and understanding place-value concepts. Consider, for example, 28 + 7. Using the Make 10 strategy, students can add 2 up to 30 and then 5 more. This strategy can be extended to make 100.

Up Over 10

Thirty-six facts have sums greater than 10, and all of those facts can be solved by using the Up Over 10 strategy. That makes this a very useful strategy. Before using this strategy, be sure that students have learned to think of the numbers 11 to 18 as 10 and some more. Surprisingly many third-grade students have not constructed this relationship.

In this approach, students use their known facts that equal 10 and then add the rest of the remaining addend on to the 10. For example, students solving 6 + 8 can start with the larger number and see that 8 is 2 away from 10; therefore, they take 2 from the 6 to get 10 and then add on the remaining 4 to get 14. This process is also called Break Apart to Make Ten (or BAMT) (Sarama & Clements, 2009).

Activity 9.2 MOVE IT, MOVE IT

BLM

Give students a mat with two ten-frames. Flash cards are placed next to the ten-frames, or a fact can be given orally. The students should first model each number in the two ten-frames with counters to represent the problem (9 + 6 would mean covering nine places on one frame and six on the other). Ask students to move it—that is, to decide a way to group the counters to show (without counting) what the total is. Ask students to explain what they did and connect it to the new equation. For example 9 + 6 may become 10 + 5 by moving the counter up to the first ten-frame. Emphasize strategies that are working for students, such as Make 10 or Up Over 10 (see Blackline Master 1). After students have found a total, have students share and record the equations. Students who are still using counting strategies or students with disabilities may need additional experience or one-on-one time working on this process.

The Up Over 10 strategy is often not emphasized in U.S. textbooks or classrooms (Henry & Brown, 2008) but it is heavily emphasized in high-performing countries (Korea, China, Taiwan, and Japan) in which students learn facts sooner and more accurately. A recent study found that the Make 10 strategy contributed more to developing fluency with Up

Over 10 facts (e.g., 7 + 8) than using doubles (even though using doubles had been emphasized by teachers and textbooks in the study). Moreover, this strategy can be later applied to adding "up over" 20, 50, or other benchmark numbers. Thus, the long-term utility of this reasoning strategy deserves significant attention.

Activity 9.3 FRAMES AND FACTS

Make little ten-frame cards (Blackline Masters 2 and 3) and display them to the class on a projector. Show an 8 (or 9) card. Place other cards beneath it one at a time as students respond with the total. Have students say orally what they are doing. For 8 + 4, they might say, "Take 2 from the 4 and put it with 8 to make 10. Then 10 and 2 left over is 12." Move to harder cards, like 7 + 6. Ask students to record each equation. Especially for students with disabilities, highlight how they should explicitly think about filling in the little ten-frame starting with the higher number. Show and talk about how it is more challenging to start with the lower number as a counterexample.

Doubles and Near-Doubles

There are only 10 doubles facts, and only seven of these have addends of 3 or greater. However, these seven facts provide useful anchors for other facts. One way to begin developing the doubles is by using a children's literature connection, *Two of Everything* (Hong, 1993). This Chinese folktale is the story of a couple (the Haktaks) who finds a magic pot that doubles everything that is put inside. Using that story as a context, a simple doubling machine can be drawn on the board in the shape of a pot, or students can have a mat with a pot shape. First, concrete items of a given number are deposited in the pot with the question "What amount comes out?" Then cards can be made with an input number on one card and the students have to write the output on another card as it goes through the doubling pot or function machine. A pair of students or a small group can use input/output machines, with one student suggesting the input and the other(s) stating the output.

Activity 9.4 CALCULATOR DOUBLES

For this activity, students work in pairs with a calculator. The students first make their calculator into a "double maker" by pressing 2 [×] into the calculator. Then one student says a double fact, for example "seven plus seven." The student with the calculator presses 7 [×] on the calculator, says what the double is, and then presses [=] to see the double (14) on the display. The students then switch roles. For ELLs who are just learning English, invite them to say the double in their native language and in English. (Note that the calculator is also useful for practicing the + 1 and + 2 facts.)

+	0	1	2	3	4	5	6	7	8	9
0	0	1								
1	1	2	3							
2		3	4	5						
3			5	6	7					
4				7	8	9				
5					9	10	11			
6						11	12	13		
7							13	14	15	
8								15	16	17
9									17	18

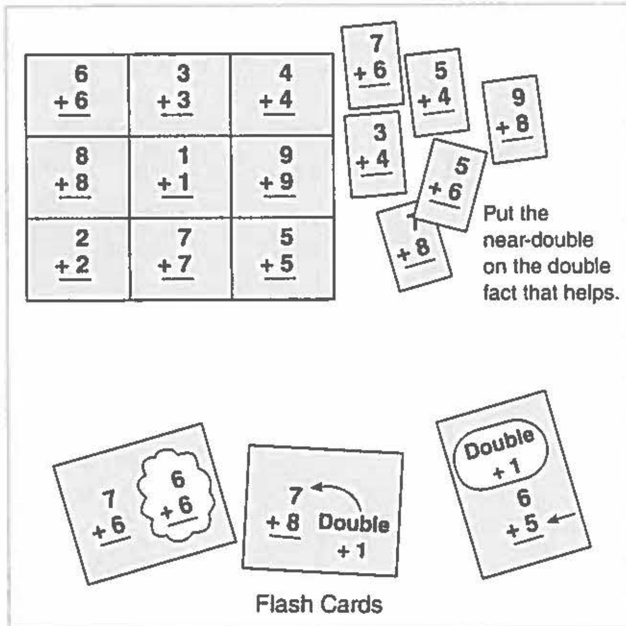
The doubles are also useful starting points for another set of facts often referred to as the *near-doubles:* facts such as 6 + 7 or 5 + 4, for which the addends are only one apart. These facts are shown here. The strategy for these doubles-plus-one or doubles-minus-one facts uses a known fact to derive an unknown fact. The strategy is to double the smaller number and add 1, or to double the larger and then subtract 1. Be sure students know the doubles before you focus on this strategy.

Something to be looking for is that some students with weak number concepts might apply double-plus-one incorrectly by beginning with the larger addend rather than the smaller. For example, they may use double 7 plus 1 for 7 + 6. Therefore, it is a good idea to focus especially on the doubling of the smaller addend. To counteract this error, write approximately 10 near-doubles facts on the board. Vary which addend is smaller. Have students solve problems independently, write the answers, and then discuss their ideas for efficient methods of answering these facts. Make sure they share their thinking so others can benefit from their decision-making process. Some students find it easy to extend the idea of the near-doubles to double-plus-two.

Activity 9.5 ON THE DOUBLE!

Create a display that illustrates the doubles (see Figure 9.2). Prepare cards with near-doubles (e.g., 4 + 5). Ask students to find the double that could help them solve the fact they have on the card and place it on that spot. Ask students if there are other doubles that could help.

Figure 9.2 Near-doubles facts.

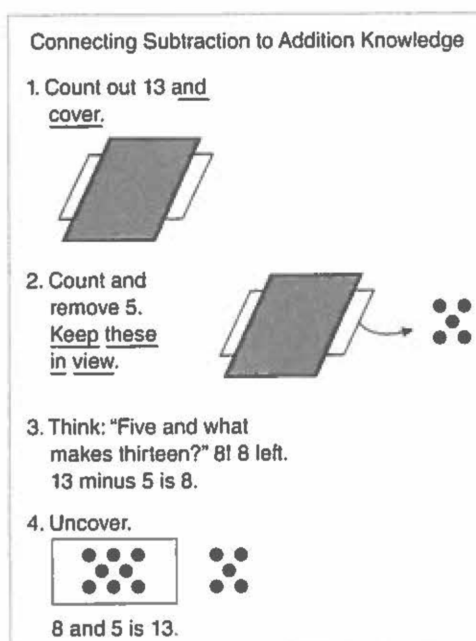


Reasoning Strategies for Subtraction Facts

Subtraction facts prove to be more difficult than addition. This is especially true when students have been taught subtraction through a *count-count-count* approach—that is, for 13 − 5, count 13, count off 5, count what's left. Counting is the first phase in reaching basic fact mastery, but unfortunately many sixth, seventh, and eighth graders are still counting.

Without opportunities to learn and use reasoning strategies, students will continue to rely on inefficient counting strategies to come up with subtraction facts, which is a slow and often inaccurate approach. Therefore, spend sufficient time working on the reasoning strategies outlined here to help students move to phase 2 and eventually on to mastery (phase 3).

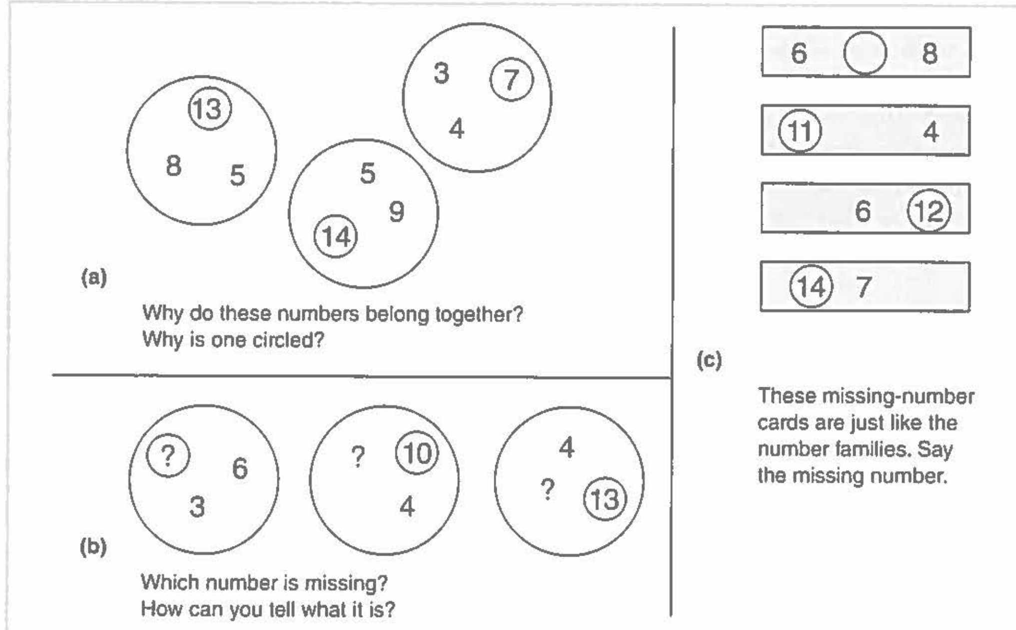
Figure 9.3 Using a think-addition model for subtraction.



Subtraction as Think-Addition

In Figure 9.3 subtraction is modeled in such a way that students are encouraged to think, "What adds to this part to make the total?" When done in this think-addition manner, students use known addition facts and the inverse relationship of addition to subtraction to produce the unknown quantity or part. If this important relationship between parts and wholes—between addition and subtraction—can be made, subtraction facts and two- and three-digit subtraction problems will be much easier (Peltenburg, van den Heuvel-Panhuizen, & Robitzsch, 2012). When students see 9 − 4, you want them to think spontaneously, "Four and what adds to nine?" By contrast, observe a third-grade student who struggles with this fact. The idea of thinking addition never occurs to the student. Instead, the student counts back from 9 and may or may not end up with 5 as the answer. The value of think-addition to solve subtraction problems cannot be overstated. This means that it is essential that addition facts be mastered first.

Figure 9.4 Introducing missing number cards.



Story problems that promote think-addition are those that sound like addition but have a missing addend: join-start unknown; join-change unknown; and part-part-whole-part unknown (see Chapter 8 for more on these problem structures). Consider this problem:

Jack had 5 fish in his aquarium. Grandma gave him some more fish. Then he had 12 fish. How many fish did Grandma give Jack?

Standards for Mathematical Practice

7 Look for and make use of structure

Notice that the action is join, which suggests addition. Students should think, "*Five and how many more adds up to 12?*" In the discussion in which you use problems such as this, your task is to connect this thought process with the subtraction fact 12 − 5. Students may use an Up Over 10 strategy to solve this problem, just as they did with addition facts ("It takes 5 to get to 10 and 2 more to 12 is 7").

Activity 9.6 MISSING NUMBER CARDS

Show students families of numbers with the sum circled as in Figure 9.4a. Ask why they think the numbers go together and why one number is circled. When this number family idea is understood, draw a different card and cover one of the numbers with your thumb, saying, "What's missing?" Ask students how they figured it out. After your modeling, students can do this with partners. Alternatively, you can create cards with one number replaced by a question mark, as in Figure 9.4b.

BLM

When students understand this activity, explain that you have made some missing-number cards based on this idea, as in Figure 9.4c (see Blackline Master 17). Ask students to name the missing number and explain their thinking. Differentiate for students with disabilities by strategically selecting a cluster that emphasizes a particular strategy (see Figure 9.5).

Formative Assessment Note

If you suspect students have not yet mastered their subtraction facts, prepare a page of them and a corresponding page of addition facts. To make the correlation between the addition and subtraction facts easier for you to see, match the facts on the two pages by putting the addition fact 5 + 4 in the same position (row and column) on the addition page as 9 − 4 is on the subtraction page.

Ask students to respond only to those facts that they know quickly without having to resort to counting. Explain that you only want to find out what they know so that you can help them with those facts they have yet to master.

If you find that they have not mastered all or nearly all of their addition facts, then that is the place to begin. The paired addition and subtraction facts may give you evidence as to whether students are using addition facts to respond to subtraction facts. If addition facts are known but subtraction facts are not known, then your task is to help students develop a think-addition approach.

Stop and Reflect

Look at the three subtraction facts shown here and try to reflect on what thought process you use to get the answers. Even if you "just know them," think about what a possible approach might be. ■

$$\begin{array}{r}14\\-9\\\hline\end{array}\qquad\begin{array}{r}12\\-6\\\hline\end{array}\qquad\begin{array}{r}15\\-6\\\hline\end{array}$$

Down Over 10

You may have applied a think-addition strategy for any of the problems in the Stop and Reflect. Or, you may have started with the 14 and counted down to 10 (4) and then down 1 more to 9, for a total difference of 5. This reasoning strategy is called Down Over 10. If you didn't already use this strategy, try it with one of the examples.

This reasoning strategy is a derived fact strategy because students use what they know (that 14 minus 4 is 10) to figure out a related fact (14 − 5). In the same way as the Make 10 and Up Over 10 strategies, this strategy is also emphasized in high-performing countries (Fuson & Kwon, 1992) and frequently ignored in the United States. One reason this strategy is so useful is that it supports students' number sense while moving them to fact mastery.

One way to develop the Down Over 10 strategy is to write five or six pairs of facts in which the difference for the first fact is 10 and the second fact is either 8 or 9—for example, 16 − 6 and 16 − 7 or 14 − 4 and 14 − 6. Have students solve each problem and discuss their strategies. If students do not naturally see the relationship, ask them to think about how the first fact can help solve the second. Reinforce the Down Over 10 strategy, by posing story problems such as the following:

Becky had 16 stuffed animals. She gave 7 to a friend. How many does Becky have left?

BLM

Figure 9.5

Missing number cards worksheet. The blank version can be used to fill in any sets of facts you wish to emphasize (see Blackline Master 17).

Make-ten facts	Near-doubles	Two fact families (7, 8, 15) (4, 8, 12)
4 ( ) 8	5 6 ( )	4 (12)
( ) 9 6	(13) 7	(15) 8
8 7 ( )	(15) 8	(12) 4
(15) 6	5 (11)	7 8 ( )
5 (13)	7 (15)	(12) 8
8 (17)	(9) 4	(12) 8
6 ( ) 8	(17) 8	4 (12)
3 9 ( )	(11) 6	8 (15)
9 (16)	5 ( ) 4	(15) 7
( ) 6 8	3 (7)	7 ( ) 8
7 (16)	(9) 5	4 (12)
3 ( ) 9	6 (13)	( ) 4 8
8 ( ) 8	(17) 9	8 (15)

Take from the 10

This strategy is also consistently used in high-performing countries. It takes advantage of students' knowledge of the combinations that make 10, and it works for all subtraction problems in which the starting value (minuend) is greater than 10. For example, take the problem 16 − 8. Students decompose the minuend into 10 + 6. Subtracting from the 10 (because they know this fact), 10 − 8 is 2. Then they add the 6 back on to get 8. Try it on these examples:

15 − 8 =   17 − 9 =   14 − 8 =

Reasoning Strategies for Multiplication and Division Facts

Using a problem-based approach and focusing on reasoning strategies are just as important for developing mastery of the multiplication and related division facts (Baroody, 2006; Wallace & Gurganus, 2005). Multiplication facts can and should be mastered by relating new facts to existing knowledge. As with addition and subtraction facts, start with story problems and concrete materials as you develop reasoning strategies.

Teaching Tip

Although the numerical answers to 2 × 8 and 8 × 2 are the same, the calculations they mean are different: 2 × 8 means 2 groups of 8; 8 × 2 means 8 groups of 2. The model you use should help explain the calculation. Although you shouldn't mark a student as incorrect for missing this distinction, you need to be precise in your language and models as you demonstrate representations of the problem in class.

Understanding the commutative property cuts the basic facts to be memorized in half. Therefore, students should completely understand the commutative property. This can be visualized by using arrays. For example, an array to show 2 × 8 can be described as 2 rows of 8 or 8 rows of 2. In both cases, the answer is 16. For a virtual site to connect arrays to multiplication facts, go to www.haelmedia.com/OnlineActivities_txh/mc_txh3_002.html.

×	0	1	2	3	4	5	6	7	8	9
0			0							
1			2							
2	0	2	4	6	8	10	12	14	16	18
3			6							
4			8							
5			10							
6			12							
7			14							
8			16							
9			18							

Doubles

Facts that have 2 as a factor are equivalent to the addition doubles and should already be known by students by the time you introduce them to multiplication. So the goal is to help students realize that 2 × 7 is the same as double 7, but so is 7 × 2. Try story problems in which 2 is the number of sets. For example, have a calendar available and ask, "Our field trip is in 2 weeks. How many days will we need to wait?" (2 groups of 7) Later, use problems in which 2 is the size of the sets. For example have buttons available and ask, "George was making sock puppets. Each puppet needed 2 buttons for eyes. If George makes 7 puppets, how many buttons will he need for the eyes?" (7 groups of 2)

×	0	1	2	3	4	5	6	7	8	9
0						0				
1						5				
2						10				
3						15				
4						20				
5	0	5	10	15	20	25	30	35	40	45
6						30				
7						35				
8						40				
9						45				

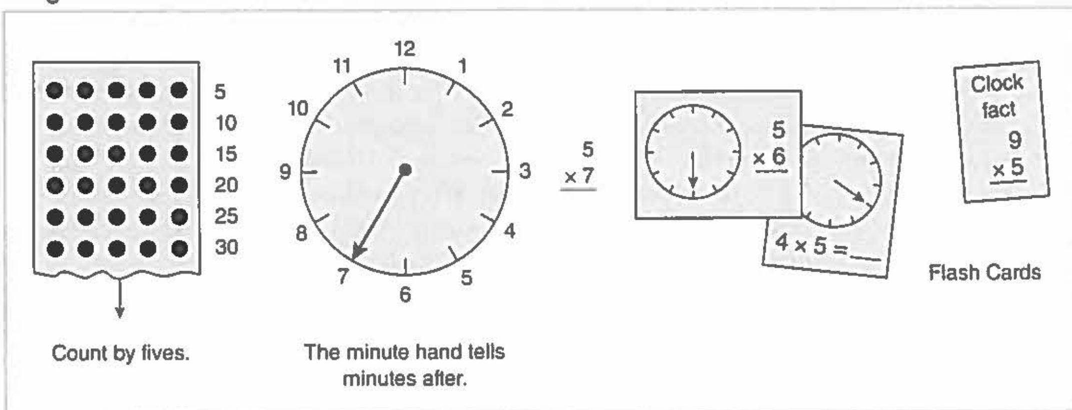
Fives

This group of facts includes all that have 5 as the first or second factor, as shown here. Practice skip counting by fives to 100. Connect counting by fives with arrays that have rows of 5 dots (see Figure 9.6). Point out that such an array with six rows is a model for 6 × 5, eight rows is 8 × 5, and so on. Connections can be made to counting minutes on a clock too.

Zeros and Ones

Thirty-six facts have at least one factor that is either 0 or 1. These facts, though seemingly easy on a procedural level, sometimes confuse students with rules they may have learned for addition. When you add zero to a number (6 + 0), it does not change the number, but 6 × 0

Figure 9.6 Fives facts.



Activity 9.7 CLOCK FACTS

Focus on the minute hand of the clock. When it points to a number, how many minutes after the hour is it? Draw a large clock face, and point to numbers 1 to 9 in random order. Students respond with the minutes after. Now connect this idea to the multiplication facts with 5. In this way, the fives facts become the clock facts.

is always zero. The $n + 1$ fact can be thought of as one more or the next number, but $n \times 1$ does not change the number. The concepts behind these facts can be developed best through story problems and an emphasis on the meaning of the operations. Alternatively to story problems, ask students to put words to the equations. For example, say that 6×0 is six groups with zero items in them (or six rows of chairs with no people in each). For 0×6, there are six in the group, but you have zero groups. For example, you worked 0 hours babysitting at \$6 an hour. Avoid rules that are strictly procedural, such as "Any number multiplied by zero is zero."

×	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2								
3	0	3								
4	0	4								
5	0	5								
6	0	6								
7	0	7								
8	0	8								
9	0	9								

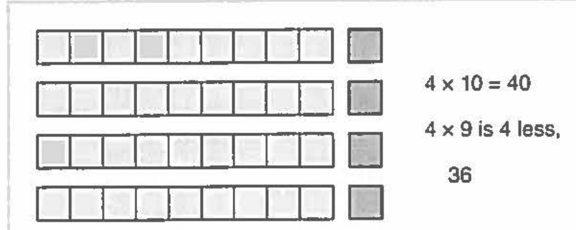
Nifty Nines

Facts with a factor of 9 include the largest products but can be among the easiest to learn due to several reasoning strategies and patterns that support students' learning. First, students can derive that 9×7 is the same as 10×7 less one set of 7, or $70 - 7$. Because students can often easily multiply by 10 and subtract from a decade value (if they've mastered their Make 10 combinations), this strategy makes sense. You might introduce a related idea by showing a set of connecting cubes (see Figure 9.7) with only the end cube a different color. After explaining that every bar has 10 cubes, ask students to find a way to figure out how many are light gray.

Second, a table of nines facts includes some interesting patterns that lead to finding the products: (1) the tens digit of the product is always one less than the "other" factor (the factor other than 9), and (2) the sum of the two digits in the product is always 9. For 7×9, 1 less than 7 is 6, and 6 and 3 makes 9, so the answer is 63. In order for students to explore and discover this pattern, ask students to record each fact for nines in order ($9 \times 1 = 9, 9 \times 2 = 18 \ldots 9 \times 9 = 81$) and write down patterns they notice. After discussing all the patterns, ask students how

×	0	1	2	3	4	5	6	7	8	9
0										0
1										9
2										18
3										27
4										36
5										45
6										54
7										63
8										72
9	0	9	18	27	36	45	54	63	72	81

Figure 9.7 Another way to think of the nines.



Standards for Mathematical Practice

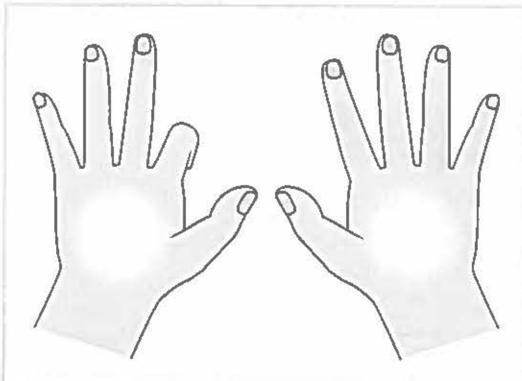
8 Look for and express regularity in repeated reasoning

these patterns can be used to figure out a product to a nines fact. (*Warning:* This strategy, grounded in the base-ten system, can be useful, but it also can cause confusion because the conceptual connection is not easy to see.) It is not, however, a rule without reason. Challenge students to think about why this pattern works. The nifty-nine pattern illustrates clearly one of the values of pattern and regularity in mathematics.

Once students have invented a strategy for the nines based on these patterns, a tactile way to help students remember the nifty nines is to use fingers—but not for counting! Here's how: Hold up both hands. Starting with the pinky on your left hand, count over to the finger that matches the factor (other than nine). For example, for 4 × 9, you move to the fourth finger (see Figure 9.8). Bend it down. Now look at the fingers – those to the left of the folded finger represent tens; those to the right represent ones. You have three tens to the left of the folded finger and six ones to the right—36 (Barney, 1970).

Figure 9.8

Nifty nines using fingers to show 4 × 9.



Using Known Facts to Derive Other Facts

Only 25 multiplication facts remain (actually fewer, due to the commutative property). These 25 facts can be learned by relating each to an already known fact or helping fact. For example, 3 × 8 is connected to 2 × 8 (double 8 and 8 more). The 6 × 7 fact can be related to either 5 × 7 (5 sevens and 7 more) or to 3 × 7 (double 3 × 7). The helping fact must be known, and the ability to do the mental addition must also be there. For example, to go from 5 × 7 = 35 and then add 7 for 6 × 7, a student must be able to efficiently add 35 and 7.

BLM

Arrays are powerful models for these strategies. Provide students with copies of the 10 × 10 dot array (Figure 9.9) (see also Blackline Master 18). The lines in the array make counting the dots easier and often suggest the use of the easier fives facts as helpers. For example, 7 × 7 is 5 × 7 plus double 7, or 35 + 14.

Knowing the doubles strategy is very effective in helping students learn difficult facts (Flowers and Rubenstein, 2010–2011). The Double and Double Again strategy shown in Figure 9.10a is applicable to all facts with 4 as a factor. For example 4 × 6 is the same as 2 × 6 doubled. But for 4 × 8, double 16 is also a difficult addition. Help students with this by noting, for example, that 15 + 15 is 30, and 16 + 16 is 2 more, or 32. Adding 16 + 16 on paper defeats the development of efficient reasoning.

×	0	1	2	3	4	5	6	7	8	9
0										
1										
2										
3				9	12		18	21	24	
4				12	16		24	28	32	
5										
6				18	24		36	42	48	
7				21	28		42	49	56	
8				24	32		48	56	64	
9										

The Double and One More Set strategy shown in Figure 9.10b is a way to think of facts with 3 as one factor. With an array or a set picture, the double part can be circled, and it is clear that there is one more set. Two facts in this group involve more difficult mental additions: 3 × 8 and 3 × 9. Using doubling and one more, you can generate any fact.

The Half Then Double strategy can be used if either factor is even. Select the even factor, and cut it in half as shown in Figure 9.10c. If the smaller fact is known, that product is doubled to get the new fact.

Students often use a Close Fact strategy when they add one more set to a known fact, as shown in Figure 9.10d. For example, think of 6 × 7 as 6 sevens. Five sevens is close: That's 35. Six sevens is one more seven, or 42. When using 5 × 8 to help with 6 × 8, the language "6 groups of eight" or "6 eights" is very helpful in remembering to add 8 more and not 6 more. This Close Fact reasoning strategy has no limits—it can be used for any multiplication fact. It also reinforces students' number sense and relationships between numbers. Asking students whether they know a nearby fact to derive the new fact over time will help make this mental process become automatic for students.

Standards for Mathematical Practice

7 Look for and make use of structure

Formative Assessment Note

Use word problems as a vehicle for assessing harder facts in a one-on-one diagnostic interview. Consider this problem: *Connie put her old crayons into bags of 7. She was able to make 8 bags with 3 crayons left over. How many crayons did she have?* Or *Carlos and Jack kept their baseball cards in albums with 6 cards on each page. Carlos had 4 pages filled, and Jack had 8 pages filled. How many cards did each boy have?* (Do you see the Half Then Double strategy?)

As the student works to get an answer, encourage her to talk about the strategies she is using. Ask her if she can solve it another way. This push adds to the benefit of the assessment by seeing what methods your student can pull from. Remember, students with disabilities may need arrays and pictures of sets or groups to help interpret the information from the problems and support their thinking about multiplication facts and relationships.

Activity 9.8 IF YOU DIDN'T KNOW

Pose the following task: "If you did not know the answer to 8 × 7 [or any fact that you want students to think about], how could you figure it out without counting?" Encourage students to come up with more than one way, hopefully using the strategies suggested above. ELLs and reluctant learners benefit from first sharing their ideas with a partner and then with the class.

BLM

Figure 9.9

An array is a useful model for developing strategies for the hard multiplication facts (see Blackline Master 18).

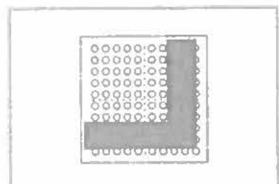
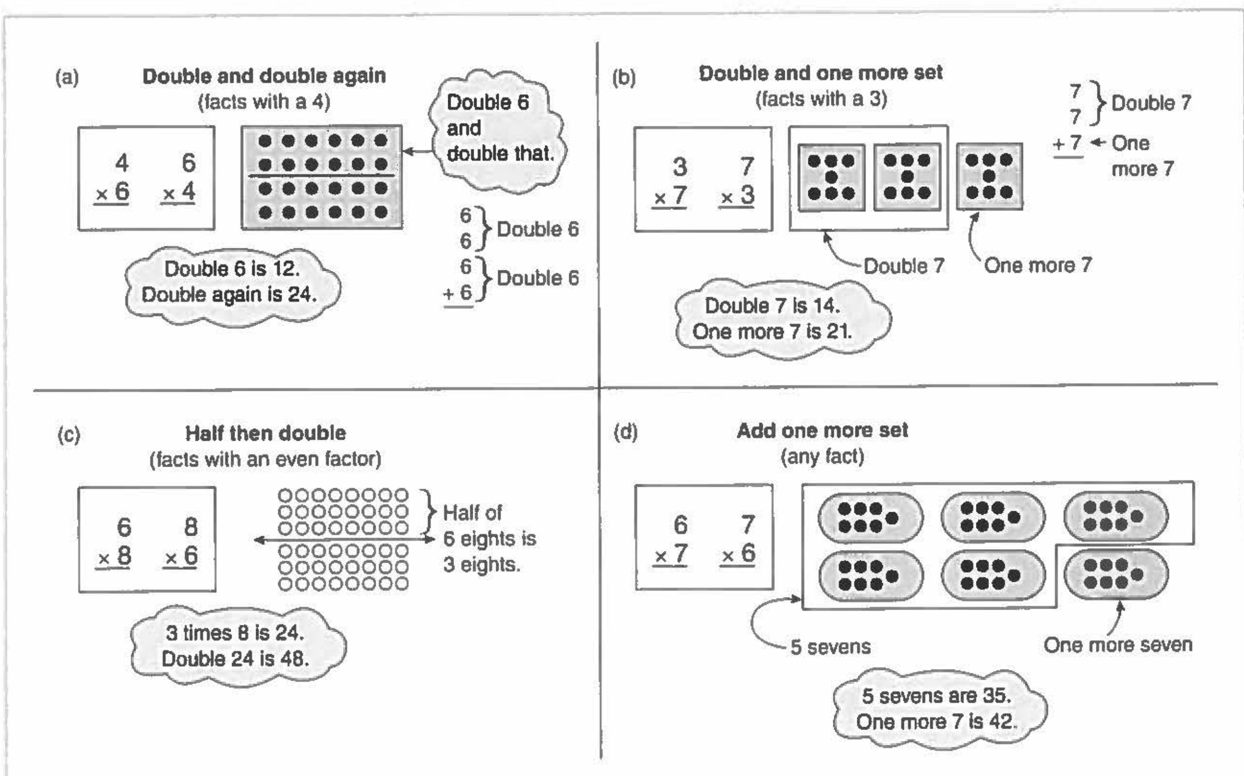


Figure 9.10

Reasoning strategies for using a known fact to derive an unknown fact.



Division Facts

Division fact mastery is dependent on the inverse relationship of multiplication and division. For example, to solve 36 ÷ 9, you tend to think, "Nine times what is thirty-six?" In fact, because of this relationship, the reasoning strategies for division are to (1) think multiplication and then (2) apply a known multiplication fact, as needed. Story problems continue to be a key vehicle to develop this connection.

Exercises such as 50 ÷ 6 might be called *near facts*. Divisions with remainders are much more prevalent in real-life situations than division facts or division without remainders. To determine the answer to 50 ÷ 6, most people mentally review a short sequence of multiplication facts, comparing each product to 50: 6 times 7 (low), 6 times 8 (close), 6 times 9 (high). It must be 8. That's 48 and 2 left over. Students should be able to do these near-fact problems mentally and with reasonable speed.

Activity 9.9 HOW CLOSE CAN YOU GET?

To practice near facts, try this exercise. Help students develop the process of going through the multiplication facts as was just described. This can be a game for small groups or an activity with the full class.

4 × □ → 23, ______ left over

Find the largest factor without going over the target number.

7 × □ → 52, □ left over

6 × □ → 27, □ left over

9 × □ → 60, □ left over

Mastering the Basic Facts

The *Common Core State Standards* precisely state that students will know their facts from memory. This is a result of repeated experiences with reasoning strategies, not because of time spent memorizing. This is an important distinction to make in mastering the facts (phase 3). Fortunately, there is quite a bit known about helping students develop fact mastery, and it has little to do with quantity of drill or drill techniques.

Drill in the absence of accomplishing success at previous phases has repeatedly been demonstrated as ineffective. However, drill strengthens memory and retrieval capabilities (Ashcraft & Christy, 1995). Students must master the basic facts (develop quick recall). Students who continue to struggle with the facts often fail to understand higher mathematics concepts; their cognitive energy gets pulled into computation when it should be focusing on the more sophisticated concepts being developed (Forbringer & Fahsl, 2010).

Effective Drill

Drill can only help students get faster at what they already know. This repetitive non-problem-based activity is appropriate once students have learned the desired concepts and are effectively using reasoning strategies and flexible approaches, but greater speed and accuracy are needed. Remember, a little goes a long way. Too often drill includes too many facts too quickly, and students become frustrated, overwhelmed, and unmotivated. Five multiplication problems targeted to a particular student can be as useful in assessing student understanding as 25 problems.

Therefore, not much is gained from the additional 20 problems. Also, when students are making mistakes, more drill and practice is not the solution—identifying and addressing misconceptions is far more effective. Because students progress at different paces—gifted students tend to be good memorizers, whereas students with intellectual disabilities have difficulty memorizing (Forbringer & Fahsl, 2010)—drilling using the same problems for all students rarely makes sense.

When working on moving students to phase 3 (know from memory), as with strategies, identify a group of facts that are related. Flash cards, for example, are more effective if they are not covering all facts and strategies but are focused on a select group that the student is ready to memorize. For example, if given a stack of $\times 1$ facts, some students will quickly learn these facts, noting the generalizations, but for some students—in particular, students with disabilities—more discussion and illustration are often needed. Because many students need multiple experiences, it is important that instruction is differentiated and engaging.

When students are still struggling with gaps in their knowledge of facts, make sure your temptation to drill is warranted. Before committing to this solution, ask yourself two questions: Will drill build understanding? How is this affecting the student's disposition toward learning? What students often learn from more drill is "Math is full of rules that I don't understand," which leads to not liking mathematics and believing they are not good at it. In reality, when a student is making errors on a procedure, it is usually a conceptual issue (as in misconception). When the problem is conceptual, remediation should include dropping back to activities that strengthen the student's conceptual knowledge.

There is little doubt that strategy development and general number sense (number relationships and operation meanings) are the best contributors to fact mastery. Drill in the absence of these factors has repeatedly been demonstrated as ineffective. However, the positive value of drill should not be completely ignored. Drill of nearly any mental activity strengthens memory and retrieval capabilities.

technology note

One important use of technology is in differentiated drill, such as that found in Fun 4 the Brain (www.fun4thebrain.com) and Math Fact Café (www.mathfactcafe.com). These free online programs work to help all students develop fluency with math facts. In short sessions that are customized for individual learners, the software allows you to differentiate instruction based on fact families. Students have the opportunity to earn electronic rewards and then move on to more difficult exercises.

Games to Support Basic Fact Mastery

Playing games and doing activities in which students can choose from the collection of reasoning strategies discussed in this chapter will allow them to become more adept at selecting strategies and more fact fluent. Games and activities provide low-stress approaches to practicing basic facts while helping students move toward quick recall. In addition, games increase student involvement, encourage student-to-student interaction, and improve communication—all of which are related to improved academic achievement (Forbringer & Fahsl, 2010; Kamii & Anderson, 2003; Lewis, 2005).

As noted previously, focus on related clusters of facts and target what individual students need to practice. Also, encourage students to self-monitor—they can create their own game board or game that includes the facts they are working to master.

Table 9.1 offers some ideas for how classic games can be adapted to focus on basic fact mastery, as well as how each can be differentiated.

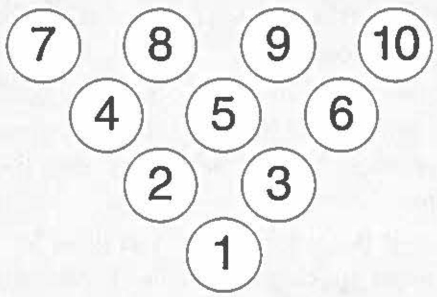
Table 9.1 Classic Games Adapted to Basic Fact Mastery

Classic Game	How to Use It with Basic Fact Mastery	Suggestions for Differentiation
Bingo	Each bingo card has a fact problem (e.g., 2 × 3) in each box. The same fact will be on multiple bingo cards but in different locations on each card. You will call out an answer (e.g., 6), and the students will find a matching problem (or more than one problem) on their card.	Create bingo boards that focus on different clusters of facts (e.g., doubles or doubles + 1 on some boards, and Up Over 10 on other boards). Be sure that the answers you call out are an even mix of the clusters so that everyone has the same chance to win.
Concentration	Create cards that have a fact problem (e.g., 3 × 5) on one half and the answer (e.g., 15) on the other half. Shuffle the cards and turn them face-down in a 6 × 4 grid. (If you like, you can make the grid larger to use more cards.)	Select cards that focus on a particular cluster of facts (e.g., + 1 and × 5 facts) for each round of the game. Multiple groups can play the game simultaneously—each group will use the parts of the deck that contain the facts they are working on. Also, consider making cards that show the ten-frames below the numbers to help provide a visual for students.
Dominoes	Create (or find online) dominoes that have a fact on one end and an answer (not to that fact) on the other end. Each student gets the same number of dominoes (around eight). On his or her turn, they can play one of the dominoes in their hand only if they have an answer or a fact that can connect to a domino on the board.	As with other games, select the dominoes that focus on particular clusters of facts.
Four in a Row	Create a 6 × 6 square game board with a sum (or product) written on each square. Below, list the numbers 0 through 9. Each of the two players has counters of a different color to use as their game pieces. On the first turn, Player 1 places a marker (paper clip) on two addends/factors and then gets to place his or her colored counter on the related answer. (If you have repeated the same answer on different squares of the board, the player only gets to cover one of them.) Player 2 can only move one paper clip and then gets to place his or her colored counter on the related answer. The first player to get four in a row wins.	Rather than list all the values below the chart, just list the related addends or factors. For example, use 1, 2, 6, 7, 8, 9 if you want to work on + 1 and + 2, or use 3, 4, 5, 6 if you are working on these multiplication facts.
Old Maid (retitled as Old Dog)	Create cards for each fact and each answer. Add one card that has a picture of an old dog (or use your school mascot). Shuffle and deal cards. On each player's turn, the player draws from the person on his or her right, sees whether that card is a match to a card in his or her hand (a fact and its answer), and, if so, lays down the pair. Then the person to the left draws from him or her. Play continues until all matches are found and someone is left with the Old Dog. Winner can be the person with (or not with) the Old Dog, or the person with the most pairs.	See Concentration (above).

Consider the following activity that engages students in creatively applying all four operations.

Activity 9.10 BOWL A FACT

In this activity (suggested by Shoecraft, 1982), you draw circles placed in a triangular fashion to look like bowling pins, with the front circle labeled 1 and the others labeled consecutively through 10. For culturally diverse classrooms, be sure that students are familiar with bowling. (If they are not, consider showing an online video clip).



Take three dice and roll them. Students use all three numbers on the three dice to come up with equations that result in answers that are on the pins. For example, if you roll 4, 2, and 3, they can "knock down" the 5 pin with $4 \times 2 - 3$. If they produce equations using these three numbers to knock down all 10 pins, they get a strike. If not, they roll again and see whether the three new numbers used in equations can knock the rest down for a spare. The pins that are left standing are added to get the score. Low score wins. After demonstrating the game, students can play in small groups.

Activity 9.11 SALUTE!

Place students in groups of three and give each group a deck of cards. (Omit face cards and use aces as ones.) Two students draw a card without looking at it and place it on their foreheads facing outward so the others can see it. The student without a card tells the product of the two cards. The first of the other two students to correctly say the factor on their forehead wins the round. For ELLs, students with disabilities, or reluctant learners, speed can increase anxiety and inhibit participation. You can remove speed of response by having the two students write down the card they think they have (within five seconds) and rewarding them one point if they are correct. This activity can be differentiated by including only certain cards (e.g., multiplication facts using only the numbers 1 through 5). The game can also be played with addition facts for students who still need support.

Fact Remediation

Students who have not mastered their basic facts by the fifth grade are in need of something other than more drill. They have certainly seen and practiced facts countless times in previous grades. There is no reason to believe that the drills *you* provide will somehow be more effective than last year's. These students need something better. The following key ideas can guide your efforts to help these older students.

1. *Recognize that more drill will not work.* Students' fact difficulties are due to a failure to develop or to connect concepts and relationships, not a lack of drill. At best, more drill will provide temporary results. At worst, it will cause negative attitudes about mathematics.

2. *Inventory the known and unknown facts for each student in need.* Identify each student's fact profile—what facts are known quickly and comfortably and which are not. Fifth- and sixth-grade students can do this diagnosis for you. Provide sheets of all facts for one operation in random order and have the students circle the facts they are hesitant about and answer all others. Caution them that finger counting or making marks in the margin is not permitted.

3. *Diagnose strengths and weaknesses.* Find out what students do when they encounter one of their unknown facts. Do they count on their fingers? Add up tic marks or numbers in the margins? Guess? Try to use a related fact? Write down times tables? Are they able to use any of the relationships suggested in this chapter? Conduct a 10-minute diagnostic interview with each student in need. Simply pose unknown facts and ask the student how he or she approaches them. Note the connections that are already there.

4. *Provide hope.* Students who have experienced long-term difficulty with fact mastery often believe that they cannot learn facts or that they are doomed to finger counting forever. Let these students know you will provide some new ideas that will help them. Take that burden on, and spare them the prospect of more defeat.

5. *Build in success.* As you begin a well-designed fact program for a student who has experienced failure, be sure that successes come quickly and easily. Begin with easy strategies, and introduce only a few new facts at a time. Exposure to five facts in a three-day period will provide more success than introducing 15 facts in a week. Success builds success! Point out to students how one strategy is all that is required to learn many facts. Use fact charts to show what set of facts you are working on. It is surprising how the chart quickly fills up with mastered facts. Keep reviewing newly learned facts and those that were already known. Success feels good and failures are not as apparent. Short practice exercises can be designed as homework. Explain strategies and build them into the exercises. At the end of the exercises, have students write about which ideas are helpful and which are not. Use this information to design the next exercise.

Your extra effort beyond class time can be motivating to a student to make some personal effort on his or her own time. During class, these students should continue to work with all students on the regular curriculum. You must believe and communicate to these students that the reason they have not mastered basic facts is not a reflection of their abilities. With efficient strategies and individual effort, success will come. Believe!

What to Do When Teaching Basic Facts

Here are important reminders about effectively teaching the basic facts. This is such an important life skill for all learners that it is imperative that you, as a teacher, use what research suggests are the most effective practices. The following list of recommendations support the development of quick recall of the basic facts.

1. *Ask students to self-monitor.* The importance of this recommendation cannot be overstated. Across all learning, having a sense of what you don't know and what you need to learn is important. It certainly holds true with memorizing facts. Students should be able to identify which facts are difficult for them and continue to work on reasoning strategies to help them derive those facts.

2. *Focus on self-improvement.* This point follows from self-monitoring. If you are working on improving students' quickness at recalling facts, students should only be competing with themselves. Students can keep track of how long it took them to go through their "fact stack," for example, and then, two days later, pull the same stack and see whether they are quicker (or more accurate) than the last time.

3. *Drill in short time segments.* You can project numerous examples of double ten-frames in relatively little time. You can also have each student pull a set of flash cards, pair with another student, and go through each other's set in 2 minutes. Long periods (10 minutes or more) are not effective. Using the first 5 to 10 minutes of the day, or extra time just before lunch, can provide continued support on fact development without taking up mathematics instructional time better devoted to other topics.

4. *Work on facts over time.* Rather than do a unit on fact memorization, work on facts over months and months, working on reasoning strategies, then on memorization, and then on continued review and monitoring.

5. *Involve families.* Share the big plan of how you will work on learning basic facts with students' families. One idea is to have one or two "Take Home Facts of the Week." Ask family members to help students by using reasoning strategies when they don't know a fact.

6. *Make drill enjoyable.* There are many games (not flash cards) designed to reinforce facts that are not competitive or anxiety inducing.

7. *Use technology.* When students work with technology, they get immediate feedback and reinforcement, helping them to self-monitor. Try www.kentuckymathematics.org /resoures/pimser.asp for some ideas.

8. *Emphasize the importance of quick recall of facts.* Without creating pressure or anxiety, highlight to students that in real life and in the rest of mathematics, they will need these facts all the time—they really must learn them and learn them well. Celebrate student successes.

What Not to Do When Teaching Basic Facts

The following list describes strategies that may have been designed with good intentions but work against student recall of the basic facts.

1. *Avoid using lengthy timed tests.* When under the pressure of time, students get distracted and abandon their reasoning strategies. Students also develop anxiety, which works against learning mathematics. Having students self-monitor the time it takes them to go through a small set of facts can help with their speed and avoid the negatives of long timed tests.

2. *Don't use public comparisons of mastery.* You may have experienced bulletin boards that show which students are on which step of a staircase to mastering their multiplication facts. Imagine how the student who is on the step 3 feels when others are on step 6. Or imagine the negative emotional reaction to public competition with flash cards for the half who don't win. Adults often refer to the competitions with flash cards as the moment they started to dislike mathematics, specifically reflecting on a game called "Around the World," in which one student is pitted against another with the loser sitting down. It is great to celebrate student successes, but avoid public comparisons between students.

3. *Don't proceed through facts in order from 0 to 9.* Work on collections of facts based on the strategies and conceptual understanding, and knock out those that students know rather than proceeding in a rigid fashion by going in numerical order.

4. *Don't work on all facts all at once.* Select a strategy (starting with easier ones) and then work on memorization of that set of facts (e.g., doubles). Be sure students really know these facts before moving on. Differentiation is needed! Students should not move to new facts until one set is mastered—otherwise they will become confused and your goal for them to master all the facts will backfire.

5. *Don't move to memorization too soon.* This has been addressed throughout the chapter, but is worth repeating. Quick recall or mastery can be attained only after students are ready, meaning they have a robust collection of reasoning strategies to apply as needed.

6. *Don't use facts as a barrier to important mathematics.* Students who have total command of basic facts do not necessarily reason better than those who, for whatever reason, have not yet mastered facts. Mathematics is not solely about computation. Mathematics is about reasoning and using patterns and making sense of things. Mathematics is problem solving. There is no reason that a student who has not yet mastered all basic facts should be excluded from more advanced mathematical experiences.

7. *Don't use fact mastery as a prerequisite for calculator use.* Insisting that students master the basic facts before allowing them to use a calculator denies them important learning opportunities. For example, if your lesson goal is for students to discover the pattern (formula) for the perimeter of rectangles, then a good lesson would have students building and exploring different-shaped rectangles, recording the length, width, and perimeter, and looking for patterns. A student who has not yet developed fact fluency will be too bogged down in computation without a calculator. With a calculator, the same student can participate and hopefully attain the learning goals of the lesson. But, once students have mastered their facts, fade the use of calculators to compute basic facts. Students should consistently practice their facts to increase their fluency and number sense.

Formative Assessment Note

If there is any purpose for a timed test of basic facts, it may be for diagnostic purposes—to determine which number combinations are mastered and which need to be learned. For it to be diagnostic, the follow-up should include the teacher and the student identifying possible misconceptions or misapplication of strategies, as well as which facts are mastered and which need more practice.